CORREZIONE ESERCIZI
FENOMENI ONDULATORI

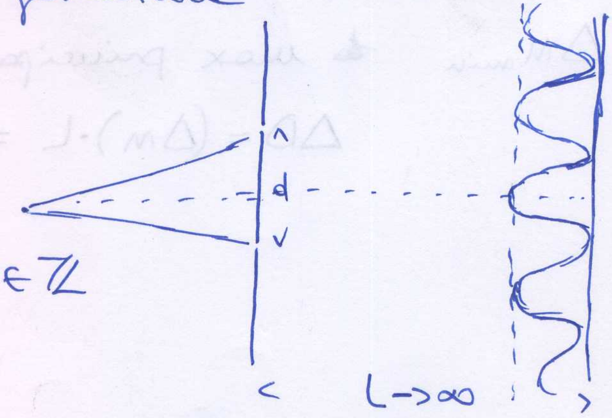
ESERCIZIO 1

Young: interferenza da doppia fenditura

$d = 0{,}1\,mm$ $\quad n_{aria} = 1$

$L = 20\,cm$ $\quad n_{H_2O} = \frac{4}{3}$

$\Delta x_{\pm 10}^{MAX} = 24\,mm$



int. costruttiva $\quad d \operatorname{sen}\vartheta = m\lambda$

int. distruttiva $\quad d \operatorname{sen}\vartheta = \left(m + \frac{1}{2}\right)\lambda \quad m \in \mathbb{Z}$

N.B. max e min equispaziati

• $\boxed{\Delta x} = \frac{\Delta x_{\pm 10}}{20} = \frac{24}{20} = 1{,}2\,mm \Rightarrow \Delta x_{0-1}$

• $\boxed{\lambda_0}$ consideriamo 1° max $m = 1$ $\quad L \gg \Delta x_{0-1} \Rightarrow \operatorname{sen}\vartheta \simeq \operatorname{tg}\vartheta$

$\operatorname{sen}\vartheta \simeq \frac{\Delta x_{0-1}}{L}$

$$d\,\frac{\Delta x_{0-1}}{L} = 1 \cdot \lambda_0 \qquad \lambda_0 = \frac{d\,\Delta x_{0-1}}{L} =$$

$$= \frac{0{,}1 \cdot 10^{-3} \cdot 1{,}2 \cdot 10^{-3}}{20 \cdot 10^{-2}} = \frac{0{,}12}{20}\,10^{-4} = 6 \cdot 10^{-7}\,m$$

• $\boxed{\lambda_0'}$ per avere le stesse frange in acqua

$d \operatorname{sen}\vartheta = m\lambda_0$ $\quad$ in generale $f_0 = f_n$

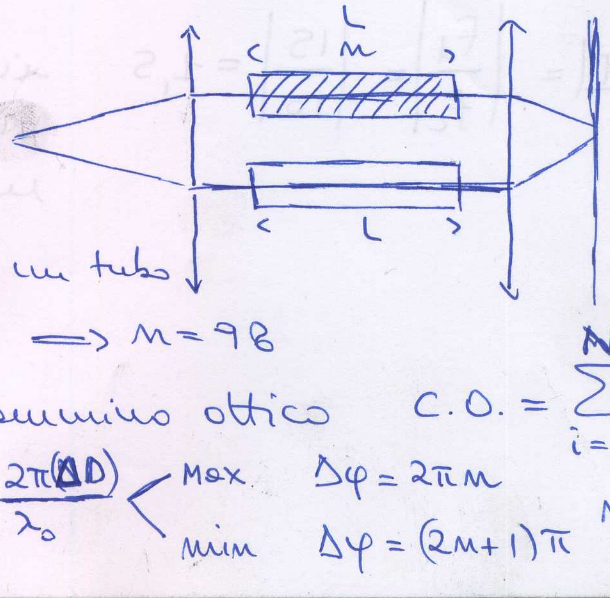
$d \operatorname{sen}\vartheta = m\,\lambda_{acqua}$ $\quad \Rightarrow \frac{c}{\lambda_0} = \frac{c}{n\lambda_n} \Rightarrow \lambda_n = \frac{\lambda_0}{n}$

$$\lambda_0 = \lambda_{acqua} = \frac{\lambda_0'}{n} \Rightarrow \lambda_0' = n\lambda_0 = \frac{4}{3} \cdot 6 \cdot 10^{-7} = 8 \cdot 10^{-7}\,m$$

ESERCIZIO 2

$\lambda = 612{,}2\,nm$ $\quad n_{gas} = ?$

$L = 20\,cm$ $\quad \Delta n_{min} = ?$



Quando si mette il gas in un tubo la figura si sposta $\quad m = 0 \Rightarrow m = 98$

conta la differenza di cammino ottico $\quad C.O. = \sum_{i=1}^{N} l_i n_i = D$

$y_1 = A \sin(k_1 x - \omega t)$

$y_2 = A \sin(k_2 x - \omega t + \varphi)$

$$\Delta\varphi = \frac{2\pi(\Delta D)}{\lambda_0} \begin{cases} \text{max} & \Delta\varphi = 2\pi m \\ \text{min} & \Delta\varphi = (2m+1)\pi \end{cases} \quad m \in \mathbb{Z}$$

$$\Delta D = n_{gas} L - 1 \cdot L = (n_{gas} - 1) \cdot L \implies \frac{2\pi (n_{gas}-1) L}{\lambda} = 2\pi \cdot 98$$

$$\implies \boxed{n_{gas}} = 1 + \frac{98\lambda}{L} = 1 + \frac{98 \cdot 612{,}2 \cdot 10^{-9}}{20 \cdot 10^{-2}}$$

$$= 1 + 2{,}999978 \cdot 10^{-4}$$

Δn_{min} max principale finisce nel 1° minimo

$$\Delta D = (\Delta n) \cdot L \implies \frac{2\pi \Delta n L}{\lambda} = \pi \implies \boxed{\Delta n} = \frac{\lambda}{2L} =$$

$$= \frac{612{,}2 \cdot 10^{-9}}{2 \cdot 20 \cdot 10^{-2}} = 1{,}53 \cdot 10^{-6}$$

ESERCIZIO 3

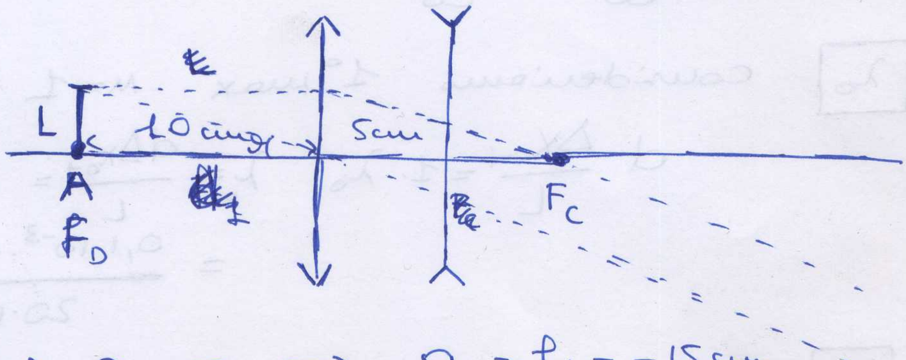
$f_c = 10\,cm$

$f_d = -15\,cm$

$p_1 = 10\,cm$

$d_1 = 5\,cm$

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$



$$p_1 = f_c \implies q_1 = \infty \implies p_2 = \infty \implies q_1 = f_d = -15\,cm$$

perciò la posizione dell'immagine coincide con quella dell'oggetto

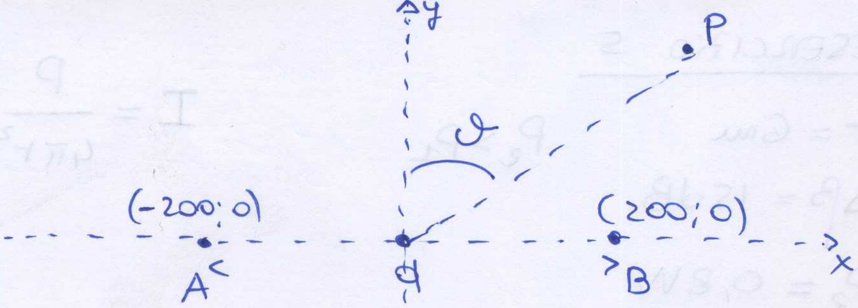
consideriamo gli angoli $\quad tg\vartheta = \frac{L}{f_c} \simeq \vartheta \qquad tg\vartheta = \frac{L'}{f_d} \simeq \vartheta \implies \frac{L}{f_c} = \frac{L'}{f_d}$

$$L' = L \cdot \boxed{\frac{f_d}{f_c}}$$

$$|I| = \left| \frac{f_d}{f_c} \right| = \left| \frac{15}{10} \right| = 1{,}5$$

immagine
virtuale
non invertita

ESERCIZIO 4



$d_{AB} = 400m$

$f = 1500 KHz$

in punti lontani

riceve segnale chiaramente $\Rightarrow$ int. costruttiva $\Delta D = m\lambda$ $\quad m \in \mathbb{Z}$

non si riceve segnale $\Rightarrow$ int. distruttiva $\Delta D = \left(m + \frac{1}{2}\right)\lambda$

$\Delta D = \overline{PA} - \overline{PB} \Rightarrow \left|\overline{PA} - \overline{PB}\right| = m\lambda$ $\quad m \in \mathbb{N}$ equazione di una iperbole con $F_1 \equiv A$ $F_2 \equiv B$ e $2a = m\lambda$

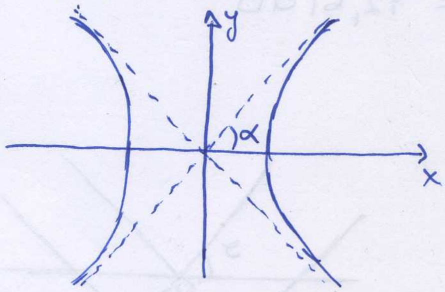
$$\lambda = \frac{c}{f} = \frac{3 \cdot 10^8}{1{,}5 \cdot 10^6} = 200m$$

$$\left|\overline{PF_1} - \overline{PF_2}\right| = 2a$$

$$a = \frac{m\lambda}{2} = 100m \qquad c = \frac{\overline{AB}}{2} = 200$$

$$b = \sqrt{c^2 - a^2} = \sqrt{200^2 - 100^2 m^2} = 100\sqrt{4 - m^2}$$

direzioni dei punti lontani di int. costruttive sono quelle degli asintoti



asintoti $y = \pm \frac{b}{a} x$

$$\pm \frac{b}{a} = \operatorname{tg}\alpha$$

$$\vartheta = \frac{\pi}{2} - \alpha \qquad \operatorname{tg}\alpha = \frac{1}{\operatorname{tg}\vartheta}$$

$$\Rightarrow \operatorname{tg}\vartheta = \pm \frac{a}{b} = \pm \frac{\cancel{100}m}{\cancel{100}\sqrt{4 - m^2}} = \pm \sqrt{\frac{m^2}{4 - m^2}}$$

$m = 0 \qquad \operatorname{tg}\vartheta = 0 \qquad \vartheta = 0°$

$m = 1 \qquad \operatorname{tg}\vartheta = \pm\sqrt{\frac{1}{3}} = \pm \frac{\sqrt{3}}{3} \qquad \vartheta = \pm 30°$

$m = 2 \qquad \operatorname{tg}\vartheta = \pm\infty \qquad \vartheta = \pm 90°$

per quelle distruttive $m \rightarrow m + \frac{1}{2}$ $\quad \operatorname{tg}\vartheta = \pm\sqrt{\frac{\left(m+\frac{1}{2}\right)^2}{4 - \left(m+\frac{1}{2}\right)^2}}$

$m = 0 \qquad \operatorname{tg}\vartheta = \pm\sqrt{\frac{\frac{1}{4}}{4 - \frac{1}{4}}} = \pm\sqrt{\frac{1}{15}} \qquad \vartheta = \pm 14{,}47°$

$m = 1 \qquad \operatorname{tg}\vartheta = \pm\sqrt{\frac{\frac{9}{4}}{4 - \frac{9}{4}}} = \pm\sqrt{\frac{9}{7}} \qquad \vartheta = \pm 48{,}59°$

ESERCIZIO 5

$r = 6m$ $\qquad P_2 > P_1$

$$I = \frac{P}{4\pi r^2} \qquad \beta = 10dB \cdot \log_{10}\frac{I}{I_0}$$

$\Delta\beta = 15dB$

$$I_0 = 10^{-12}\frac{W}{m^2}$$

$P_2 = 0,8W$

$\beta_1 = ?$ $\quad \beta_2 = ?$ $\quad \beta_{1+2} = ?$

$$\Delta\beta = \beta_2 - \beta_1 = 10dB \log_{10}\frac{I_2}{I_0} - 10dB \cdot \log_{10}\frac{I_2}{I_0}$$

$$= 10dB \cdot \log_{10}\frac{\frac{I_2}{\cancel{I_0}}}{\frac{I_1}{\cancel{I_0}}} =$$

$$= 10dB \log_{10}\frac{I_2}{I_1}$$

$$\frac{I_2}{I_1} = 10^{\frac{\Delta\beta}{10dB}} = 10^{\frac{\cancel{15}^{3}}{\cancel{10}_2}} = 31,62$$

$$I_2 = \frac{P_2}{4\pi r^2} = \frac{0,8}{4\pi \cdot 6^2} = 1,77 \cdot 10^{-3}\frac{W}{m^2} \qquad I_1 = \frac{I_2}{31,62} = 5,6 \cdot 10^{-5}\frac{W}{m^2}$$

$$\boxed{\beta_2} = 10dB \log_{10}\frac{1,77 \cdot 10^{-3}}{10^{-12}} = 92,47dB \qquad \boxed{\beta_1} = 77,48dB$$

$$\boxed{\beta_{1+2}} = 10dB \log_{10}\frac{I_1 + I_2}{I_0} = 92,61dB \qquad \beta_{1+2} \neq \beta_1 + \beta_2 \,!$$

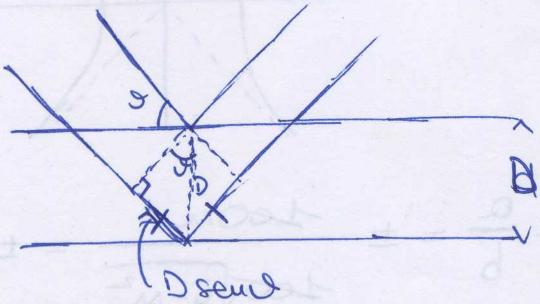
ESERCIZIO 6

raggi X $\quad \lambda = 0,166 nm$

$D = 3,14 Å$

$m = 2$ max

$\vartheta = ?$



$$2D\,\text{sen}\vartheta = m\lambda \qquad \text{Legge di Bragg}$$

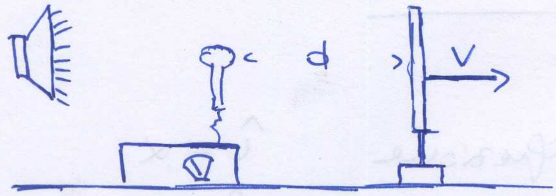
$$2D\,\text{sen}\vartheta = \left(m + \frac{1}{2}\right)\lambda$$

max $m = 2$ $\qquad \cancel{2} \cdot 3,14 \cdot 10^{-10}\,\text{sen}\vartheta = \cancel{2} \cdot 0,166 \cdot 10^{-9}$

$$\text{sen}\vartheta = \frac{1,66 \cdot \cancel{10^{-10}}}{3,14 \cdot \cancel{10^{-10}}} = 0,5286$$

$$\vartheta = \text{sen}^{-1} 0,5286 = 31,91°$$

ESERCIZIO 7



$d_1 = 22{,}5$ cm max

$d_2 = 36{,}5$ cm max

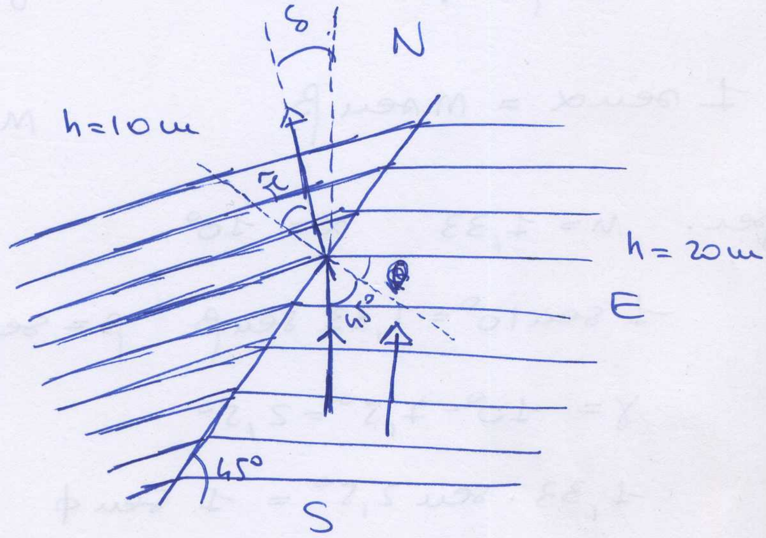
) in mezzo 10 max (incluso l'ultimo)

interferenza di onde riflesse $\Delta D = 2d \begin{cases} = m\lambda \\ = \left(m+\frac{1}{2}\right)\lambda \end{cases}$

$$2d_1 = m\lambda$$
$$2d_2 = (m+10)\lambda$$
$$\Rightarrow \cancel{2}(d_2 - d_1) = \cancel{10}^{\,5}\,\lambda$$

$$\lambda = \frac{d_2 - d_1}{5} = \frac{36{,}5 - 22{,}5}{5} = \frac{14}{5} = 2{,}8 \text{ cm}$$

ESERCIZIO 8



$$v = \sqrt{gh}$$

$$v_1 = \sqrt{20g} \qquad n_1 = \frac{v}{v_1}$$
$$v_2 = \sqrt{10g} \qquad n_2 = \frac{v}{v_2}$$

$$n_1 \operatorname{sen}\hat{i} = n_2 \operatorname{sen}\hat{r}$$
$$\frac{\cancel{v}}{v_1}\operatorname{sen}\hat{i} = \frac{\cancel{v}}{v_2}\operatorname{sen}\hat{r}$$

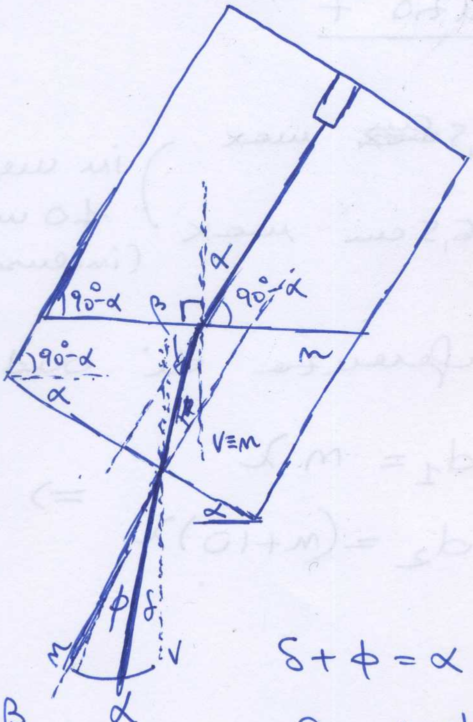
$$\operatorname{sen}\hat{r} = \frac{v_2}{v_1}\operatorname{sen}\hat{i} = \sqrt{\frac{10g}{20g}}\operatorname{sen}45° = \sqrt{\frac{1}{2}}\cdot\frac{\sqrt{2}}{2} = \frac{1}{2} \qquad \operatorname{sen}\hat{r} = \frac{1}{2} \quad \hat{r} = 30°$$

$\delta = 45° - 30° = 15°$ la direzione è di 15° da N verso W

ESERCIZIO 9

1° rifrazione $\hat{i} = \alpha$

$\hat{r} = \beta$



2° rifrazione $\hat{i} = \gamma$

$\hat{r} = \phi$

angolo rispetto alla verticale δ

$\delta + \phi = \alpha$

$$\beta + \gamma + (\pi - \alpha) = \pi \Rightarrow \gamma = \alpha - \beta$$

$\delta = \alpha - \phi$

$1 \operatorname{sen}\alpha = n \operatorname{sen}\beta$ $\qquad n \operatorname{sen}\gamma = 1 \operatorname{sen}\phi$

• per $n = 1{,}33$ $\quad \alpha = 10°$

$$1 \operatorname{sen} 10° = 1{,}33 \operatorname{sen}\beta \quad \beta = \operatorname{sen}^{-1}\left(\frac{\operatorname{sen}10°}{1{,}33}\right) = 7{,}5°$$

$$\gamma = 10° - 7{,}5° = 2{,}5°$$

$$1{,}33 \cdot \operatorname{sen} 2{,}5° = 1 \operatorname{sen}\phi \qquad \phi = \operatorname{sen}^{-1}(1{,}33 \operatorname{sen} 2{,}5°) = 3{,}33°$$

$$\delta = 10° - 3{,}33° = 6{,}67°$$

• mostrare che $\delta = k\alpha$ per α piccolo, k costante

$\operatorname{sen}\alpha = n \operatorname{sen}\beta$

$n \operatorname{sen}\gamma = \operatorname{sen}\phi$

$$\beta = \operatorname{sen}^{-1}\left(\frac{\operatorname{sen}\alpha}{n}\right) \Rightarrow \gamma = \alpha - \operatorname{sen}^{-1}\left(\frac{\operatorname{sen}\alpha}{n}\right) \Rightarrow$$

$$\Rightarrow n \operatorname{sen}\left(\alpha - \operatorname{sen}^{-1}\left(\frac{\operatorname{sen}\alpha}{n}\right)\right) = \operatorname{sen}(\alpha - \delta)$$

$n = 1{,}33$ calcolo con la calcolatrice $k \cong 0{,}667$

$$\Rightarrow \alpha - \delta = \operatorname{sen}^{-1}\left(n \operatorname{sen}\left(\alpha - \operatorname{sen}^{-1}\left(\frac{\operatorname{sen}\alpha}{n}\right)\right)\right)$$

$$\Rightarrow \delta = \alpha - \operatorname{sen}^{-1}\left(n \operatorname{sen}\left(\alpha - \operatorname{sen}^{-1}\left(\frac{\operatorname{sen}\alpha}{n}\right)\right)\right)$$

• ricordando $\operatorname{sen}\alpha \sim \alpha$ e $\operatorname{sen}^{-1}(\alpha) \sim \alpha$ per α piccolo

$$\delta \cong \alpha - \operatorname{sen}^{-1}\left(n \operatorname{sen}\left(\alpha - \operatorname{sen}^{-1}\left(\frac{\alpha}{n}\right)\right)\right) \simeq \alpha - \operatorname{sen}^{-1}\left(n \operatorname{sen}\left(\alpha - \frac{\alpha}{n}\right)\right) \simeq$$

$$\simeq \alpha - \operatorname{sen}^{-1}\left(n\left(\alpha - \frac{\alpha}{n}\right)\right) \simeq \alpha - \left(n\left(\alpha - \frac{\alpha}{n}\right)\right) = \alpha - n\alpha + \alpha = \alpha(2-n)$$

per $n = 2$ $\forall \alpha$ $\delta = 0$