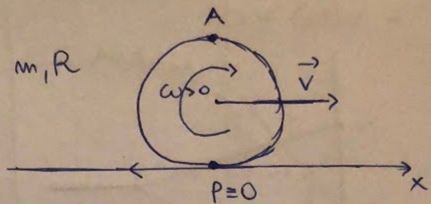
Correzione esercizi 25/01/2018

1). inizialmente $V_0 > 0$ e $\omega_0 < 0$ m, R



→ attrito nullo $V_0 = cost$
$\omega_0 = cost$

A dopo un giro $|\omega_0| = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{|\omega_0|}$ $\boxed{d} = V_0 \cdot T = \frac{2\pi V_0}{|\omega_0|}$

ora c'è attrito dinamico (coeff. μ)

→ velocità del punto P

$V_P = V_{trasl\,CM} + V_{rot} = V - \omega R > 0$ inizialmente

⇒ c'è attrito dinamico $F_a = \mu N = \mu m g$ verso sinistra
e si annulla quando $V_P = 0$

→ scrivere $V(t)$ e $u(t) = R \cdot \omega(t)$

attrito $F_a = -\mu m g \Rightarrow a = -\mu g$ $\boxed{V = V_0 - \mu g t}$

momento $\tau = +|F_a| R = I \cdot \alpha \Rightarrow \alpha = + \frac{+\mu m g R}{I} = + \frac{\mu \cancel{m} g R}{\cancel{m} R^2} = + \frac{\mu g}{R}$

cilindro cavo $I = mR^2$

$\omega = \omega_0 + \frac{\mu g}{R} t \Rightarrow \boxed{u = \omega_0 R + \mu g t}$

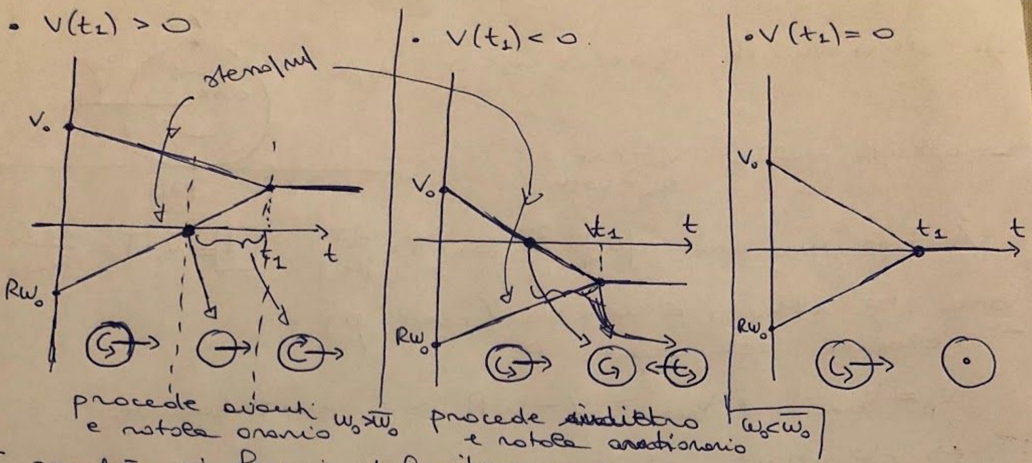
t_1 tale che $V(t_1) = u(t_1)$

$$V_0 - \mu g t_1 = \omega_0 R + \mu g t_1$$
$$2\mu g t_1 = V_0 - \omega_0 R$$
$$t_1 = \frac{V_0 - \omega_0 R}{2\mu g} > 0$$

$$\boxed{V(t_1)} = V_0 - \cancel{\mu g}\left(\frac{V_0 - \omega_0 R}{2\cancel{\mu g}}\right) = \frac{2V_0 - V_0 + \omega_0 R}{2} = \frac{V_0 + \omega_0 R}{2}$$

$$u(t_1) = \frac{V_0 + \omega_0 R}{2} \Rightarrow \boxed{\omega(t_1)} = \frac{V_0 + \omega_0 R}{2R}$$

possibili grafici



→ $\overline{\omega}_0$ perché si fermi definitivamente

$$v_0 - \mu g t = \overline{\omega}_0 R + \mu g t = 0$$

$v_0 = \mu g t$

$\overline{\omega}_0 R = -\mu g t = -v_0 \Rightarrow \boxed{\overline{\omega}_0 = -\frac{v_0}{R}}$

$v(t_0) = 0 \qquad t_0 = \frac{v_0}{\mu g}$

fino a t_0 MUA $\quad d = v_0 t_0 + \frac{1}{2} a t_0^2 = v_0 \frac{v_0}{\mu g} - \frac{1}{2}\mu g \frac{v_0^2}{\mu^2 g^2} = \frac{v_0^2}{2\mu g}$

→ Considerando $\omega_0 = 2\overline{\omega}_0$ e $t = 2t_1$, trova il lavoro fatto dall'attrito in termini di m e v_0

Energia $\quad t_0 = 0 \quad E_{c_i} = \frac{1}{2} m v_0^2 + \frac{1}{2} I \omega_0^2 = \frac{1}{2} m v_0^2 + \frac{1}{\cancel{2}} m \cancel{R^2} \cdot \cancel{4} \frac{v_0^2}{\cancel{R^2}} = \frac{5}{2} m v_0^2$

$t = 2t_1 \quad E_{c_f} = \frac{1}{2} m V^2 + \frac{1}{2} I \omega^2 = \frac{1}{2} m (v(t_1))^2 + \frac{1}{2} I (\omega(t_1))^2$

$v(2t_1) = v(t_1)$

$\omega(2t_1) = \omega(t_1)$

$= \frac{1}{2} m (v(t_1))^2 + \frac{1}{2} m R^2 (\omega(t_1))^2 =$

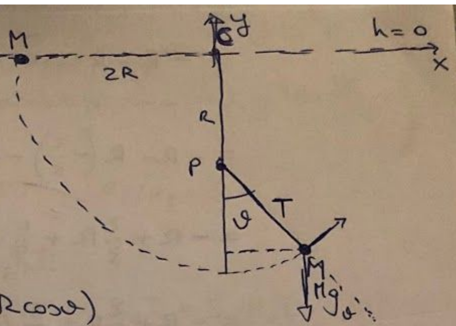
$v(t_1) = \frac{v_0 + \omega_0 R}{2} = \frac{v_0 + 2\left(-\frac{v_0}{\cancel{R}}\right)\cancel{R}}{2} = -\frac{v_0}{2} = R\omega(t_1)$

$= \frac{1}{2} m \left(+\frac{v_0^2}{4}\right) + \frac{1}{2} m \cancel{R^2} \frac{v_0^2}{4\cancel{R^2}} =$

$= \frac{1}{8} m v_0^2 + \frac{1}{8} m v_0^2 = \frac{1}{4} m v_0^2$

$\boxed{L_a} = E_{c_f} - E_{c_i} = \left(\frac{1}{4} - \frac{5}{2}\right) m v_0^2 = -\frac{9}{4} m v_0^2$

2) inizialmente $h_{pallina} = h_C$ — $h=0$



→ per quale valore di ϑ
$T = \frac{7}{2}Mg$?

energia $E_i = 0$
$E_f = \frac{1}{2}Mv^2 + Mg(-R - R\cos\vartheta)$

dinamica $T - Mg\cos\vartheta = M\frac{v^2}{R}$

$E_i = E_f$ $\frac{1}{2}\cancel{M}v^2 - \cancel{M}gR(1+\cos\vartheta) = 0$ $v^2 = 2gR(1+\cos\vartheta)$

$\Rightarrow$ $T - Mg\cos\vartheta = M \cdot 2g(1+\cos\vartheta)$

$\frac{7}{2}\cancel{Mg} - \cancel{Mg}\cos\vartheta = 2\cancel{Mg} + 2\cancel{Mg}\cos\vartheta$

$\frac{7}{2} - \cos\vartheta = 2 + 2\cos\vartheta$ $3\cos\vartheta = \frac{3}{2}$ $\cos\vartheta = \frac{1}{2}$

$\vartheta = 60°$

→ non colpisce C perché a quell'altezza $v = 0 \Rightarrow T < 0$ impossibile

→ $\tilde{\vartheta}$ per cui non si ha più traiettoria circolare

→ $T = 0$ $-Mg\cos\vartheta = \cancel{M} \cdot 2g(1+\cos\vartheta)$

$-\cos\vartheta = 2 + 2\cos\vartheta$ $\cos\vartheta = -\frac{2}{3}$ $\tilde{\vartheta} \simeq 132°$

→ quando non è più in tensione ⇒ proiettile

$v_0 = \sqrt{2gR(1+\cos\tilde{\vartheta})} = \sqrt{2gR(1-\frac{2}{3})} = \sqrt{\frac{2}{3}gR}$

$x_0 = R\,\mathrm{sen}\tilde{\vartheta}$ $v_{0x} = v_0\cos\tilde{\vartheta}$ $x = R\,\mathrm{sen}\tilde{\vartheta} + v_0\cos\tilde{\vartheta}\,t$

$y_0 = -R - R\cos\tilde{\vartheta}$ $v_{0y} = v_0\,\mathrm{sen}\tilde{\vartheta}$ $y = -R - R\cos\tilde{\vartheta} + v_0\,\mathrm{sen}\tilde{\vartheta}\,t - \frac{1}{2}gt^2$

ove verticale $x = 0$ $\Rightarrow$ $t = -\frac{R}{v_0}\mathrm{tg}\tilde{\vartheta}$

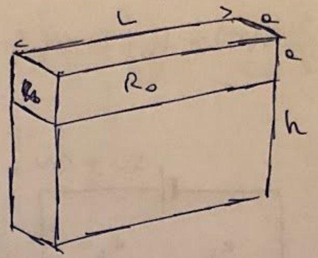
$$y = -R - R\cos\tilde{\vartheta} + v_0\,\mathrm{sen}\tilde{\vartheta}\left(-\frac{R}{v_0}\mathrm{tg}\tilde{\vartheta}\right) - \frac{1}{2}g\left(-\frac{R}{v_0}\mathrm{tg}\tilde{\vartheta}\right)^2 =$$

$$= -R - R\cos\tilde{\vartheta} - R\frac{\mathrm{sen}^2\tilde{\vartheta}}{\cos\vartheta} - \frac{1}{2}\frac{gR^2}{v_0^2}\frac{\mathrm{sen}^2\tilde{\vartheta}}{\cos^2\tilde{\vartheta}} =$$

$L = 2\,cm \qquad a = 0{,}5\,cm \qquad h = 5\,cm$

tungsteno $\qquad$ rame

$R_0 = 100\,\Omega$



T' se $\Delta V = 24\,V$

$P_{diss} = P_{conduzione}$

$$\frac{(\Delta V_0)^2}{R_0[1+\alpha(T-T_0)]} = k\,\frac{a \cdot L}{h}(T-T_0) \qquad h(\Delta V_0)^2 = k a L (T-T_0) R_0 [1+\alpha(T-T_0)]$$

$$\Rightarrow \; k a L R_0 \alpha (T-T_0)^2 + k a L R_0 (T-T_0) - h(\Delta V_0)^2 = 0$$

$$T - T_0 = \frac{-k a L R_0 \pm \sqrt{k^2 a^2 L^2 R_0^2 + 4 k a L R_0 \alpha h (\Delta V_0)^2}}{2 k a L R_0 \alpha} =$$

$$= -\frac{1}{2\alpha} + \sqrt{\frac{1}{4\alpha^2} + \frac{h(\Delta V_0)^2}{k a L R_0 \alpha}} =$$

$$= \frac{1}{2\alpha}\left(\sqrt{1 + \frac{4\alpha h (\Delta V_0)^2}{k a L R_0}} - 1\right) \sim 7{,}3\,K = 7{,}3\,°C$$

$$\Rightarrow T = 32{,}3\,°C$$

6. $\nu = 7{,}5 \cdot 10^{14}\,Hz \qquad k = \frac{2\pi}{\lambda} = \frac{2\pi\nu}{c} = \frac{\omega}{c}$

si propaga lungo x $\quad$ polarizzata linearmente

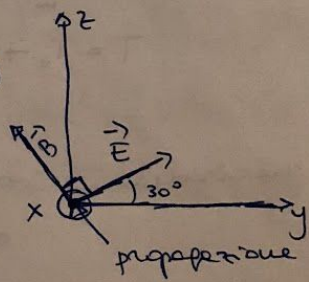
$\vec{E}$ 30° rispetto ~~all'asse x~~ al piano xy $\qquad E_0 = 103\ V/m$

$$\vec{E} = (E_0 \cos 30°\,\hat{j} + E_0 \operatorname{sen} 30°\,\hat{k}) \operatorname{sen}(kx - \omega t)$$

$$= \left(103 \frac{\sqrt{3}}{2}\,\hat{j} + 103 \cdot \frac{1}{2}\,\hat{k}\right) \operatorname{sen}\left(\frac{\omega}{c}x - \omega t\right)$$

$$= (51{,}5\sqrt{3}\,\hat{j} + 51{,}5\,\hat{k}) \operatorname{sen}\left(2\pi\nu\left(\frac{x}{c} - t\right)\right)$$

$$= (51{,}5\sqrt{3}\,\hat{j} + 51{,}5\,\hat{k}) \operatorname{sen}\left(15\pi \cdot 10^{14}\left(\frac{x}{3 \cdot 10^8} - t\right)\right)$$

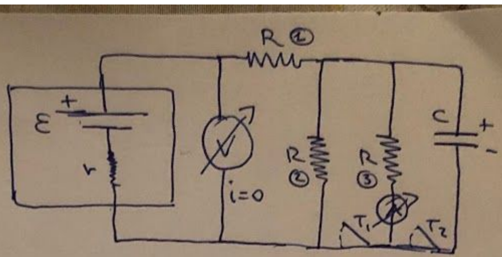


$\vec{E} = \vec{B} \times \vec{c}$ $\qquad B_0 = \frac{E_0}{c} = \frac{103}{3 \cdot 10^8} = 34{,}\bar{3} \cdot 10^{-8}\,T$

direzione di $\vec{k}$

$$\vec{B} = (34{,}3 \cdot 10^{-8} \cos 30°\,\hat{k} - 34{,}3 \cdot 10^{-8} \operatorname{sen} 30°\,\hat{j}) \operatorname{sen}\left(15\pi \cdot 10^{14}\left(\frac{x}{3 \cdot 10^8} - t\right)\right)$$

5. $R = 10\,\Omega$



prima fase : T_1 e T_2 aperti

$V = 48V$

seconda fase : chiudo T_1

$I = 1{,}5A$

→ fem e r ?

1°) I_0 erogata

$$\varepsilon - r I_0 = V$$
$$V = 2R I_0$$

2°) I_1 erogata

2//3 $\quad \frac{1}{R_{2,3}} = \frac{1}{R} + \frac{1}{R} \quad R_{2,3} = \frac{R}{2}$

$$R_{eq} = r + R + \frac{R}{2} = r + \frac{3}{2}R$$

$$I_1 = \frac{\varepsilon}{r + \frac{3}{2}R}$$

$I = \frac{I_1}{2}$ perché le resistenze sono uguali

$\Rightarrow \quad I_1 = 2I = 2 \cdot 1{,}5 = 3\,A$

$$I_0 = \frac{V}{2R} = \frac{48}{2 \cdot 10} = \frac{48}{20} = 2{,}4\,A$$

$$\begin{cases} \varepsilon - r \cdot 2{,}4 = 48 \\ \varepsilon = \left(r + \frac{3}{2} \cdot 10\right) \cdot 3 = 3r + 45 \end{cases} \Rightarrow \begin{cases} 3r + 45 - 2{,}4r = 48 \\ \varepsilon = 3r + 45 \end{cases}$$

$$\Rightarrow \begin{cases} 0{,}6r = 3 \quad \Rightarrow r = \frac{3}{0{,}6} = 5\,\Omega \\ \varepsilon = 3 \cdot 5 + 45 = 60\,V \end{cases}$$

→ $\Delta I = I_1 - I_0 = 3 - 2{,}4 = 0{,}6\,A$

→ chiudo T_2 $\quad$ C caricato prima con ε $\quad V_c = \varepsilon$

⇒ maglia esterna $\quad \varepsilon - r I_q - R I_q - V_c = 0$

$$(-r - R) I_q = 0 \qquad I_q = 0$$

$$I' = \frac{V_c}{R} = \frac{60}{10} = 6\,A$$

$$= -R - R\cos\tilde{\vartheta} - R\,\frac{1-\cos^2\tilde{\vartheta}}{\cos\tilde{\vartheta}} - \frac{1}{2}g\,\frac{R^{\cancel{2}}}{\frac{2}{3}g\cancel{R}}\;\frac{1-\cos^2\tilde{\vartheta}}{\cos^2\tilde{\vartheta}} =$$

$$= -R - R\left(-\frac{2}{3}\right) - R\,\frac{1-\frac{4}{9}}{-\frac{2}{3}} - \frac{1}{2}\cdot\frac{3R}{2}\,\frac{1-\frac{4}{9}}{+\frac{4}{9}} =$$

$$= -R + \frac{2}{3}R + \frac{5}{\cancel{9}_3}\cdot\frac{\cancel{3}}{2}R - \frac{3}{4}R\cdot\frac{5}{\cancel{9}}\cdot\frac{\cancel{9}}{4} =$$

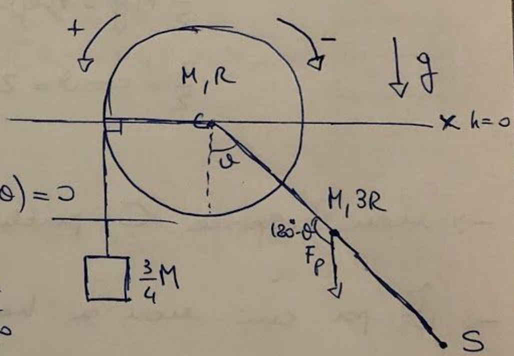
$$= -R + \frac{2}{3}R + \frac{5}{6}R - \frac{15}{16}R = \frac{-48+32+40-45}{48}R = -\frac{21}{48}R = -\frac{7}{16}R$$

→ velocità nell'istante di impatto

energia $\quad E_i = 0 \qquad E_i = E_f$

$$E_f = -\frac{7}{16}MgR + \frac{1}{2}Mv^2 \quad \Rightarrow v^2 = \frac{7}{8}Rg \quad v = \sqrt{\frac{7Rg}{8}}$$

3.



→ ϑ per cui è in equilibrio

$$\sum \tau_{1c} = +\frac{3}{4}MgR - Mg\cdot\frac{3}{2}R\cdot\mathrm{sen}(180°-\vartheta) = 0$$

$$\frac{\cancel{3}}{\cancel{4}_2}\cancel{MgR} = \frac{\cancel{3}}{2}\cancel{MgR}\,\mathrm{sen}\vartheta \quad \mathrm{sen}\vartheta = \frac{1}{2}$$

$$\vartheta = 30°$$

→ α quando si taglia la fune

$$\sum \tau_{1c} = -\frac{3}{2}MgR\,\mathrm{sen}\vartheta = I\cdot\alpha \quad \text{con } I = I_{disco} + I_{sbarra}$$

$$= I_{disco} + \left(I^{cm}_{sbarra} + Md^2\right)$$

$$\alpha = \frac{-\frac{3}{2}Mg\cancel{R}\cdot\frac{1}{2}}{\frac{14}{\cancel{4}}MR^{\cancel{2}}} = -\frac{3}{\cancel{4}}\cdot\frac{\cancel{4}}{14}\,\frac{g}{R} = \frac{-3}{14}\,\frac{g}{R}$$

$$= \frac{1}{2}MR^2 + \frac{1}{12}M(3R)^2 + M\left(\frac{3}{2}R\right)^2$$

$$= \frac{6+9+27}{12}MR^2 = \frac{\cancel{14}^7}{\cancel{4}_2}MR^2$$

→ velocità di S quando è in verticale

$$E_i = E_c + E_p = 0 + Mg\cdot\frac{3}{2}R\cos\vartheta = -\frac{3}{2}MgR\cdot\frac{\sqrt{3}}{2} = -\frac{3\sqrt{3}}{4}MgR$$

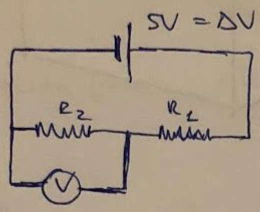
$$E_f = E_c + E_p = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}M\omega^2\left(\frac{3}{2}R\right)^2 + \frac{1}{2}\left(\frac{1}{2}MR^2 + \frac{1}{12}M(3R)^2\right) = \frac{7}{\cancel{4}}MR^2\omega^2 - \frac{3}{2}MgR$$

$$\Rightarrow -\frac{3\sqrt{3}}{4}g = \frac{7}{\cancel{4}}\omega^2R - \frac{3}{2}g \Rightarrow \omega = \sqrt{\frac{g}{R}\,\frac{6}{7}\left(1-\frac{\sqrt{3}}{2}\right)} \Rightarrow V_s = 3R\omega \quad -\frac{3}{2}MgR$$

4. $R(T) = R_0\left[1+\alpha(T-T_0)\right]$ $\quad T_0 = 25°C$ $\quad \alpha_{Tungsteno} = 4{,}5\cdot10^{-3}\,K^{-1}$

conducibilità rame $\quad k = 380\,\frac{W}{m\cdot K}$

$\rightarrow$



$V(T_0) = 2{,}5V \qquad R_1/R_2 = ?$

$5 = (R_1+R_2)I$

$R_2\cdot I = 2{,}5V \rightarrow R_1 I = 5 - R_2 I = 2{,}5V$

$$\frac{R_1 I}{R_2 I} = \frac{2{,}5}{2{,}5} \Rightarrow \frac{R_1}{R_2} = 1$$

$\rightarrow$ $R_1 = R_0\left(1+\alpha_1(T-T_0)\right)$

$R_2 = R_0\left(1+\alpha_2(T-T_0)\right)$

$$R_1+R_2 = R_0\left[2+(\alpha_1+\alpha_2)(T-T_0)\right]$$

$$I = \frac{\Delta V}{R_{TOT}} = \frac{5}{R_0\left[2+(\alpha_1+\alpha_2)(T-T_0)\right]}$$

$$\Delta V_2 = R_2 I = \frac{5R_0\left(1+\alpha_2(T-T_0)\right)}{R_0\left[2+(\alpha_1+\alpha_2)(T-T_0)\right]} = \frac{5\left(1+\alpha_2(T-T_0)\right)}{2+(\alpha_1+\alpha_2)(T-T_0)}$$

$\rightarrow$ $\Delta V_2 = 2{,}62V \qquad \alpha_1 = 2\cdot10^{-3}\,K^{-1} \qquad \alpha_2 = 4\cdot10^{-3}\,K^{-1} \qquad T = ?$

$$2\Delta V_2 + (\alpha_1+\alpha_2)\Delta V_2(T-T_0) = 5 + 5\alpha_2(T-T_0)$$

$$(T-T_0)\left[(\alpha_1+\alpha_2)\Delta V_2 - 5\alpha_2\right] = 5 - 2\Delta V_2$$

$$T = T_0 + \frac{2\Delta V_2 - 5}{5\alpha_2 - (\alpha_1+\alpha_2)\Delta V_2} = 81{,}1°C$$

$\rightarrow$ precisione: alla buona calcolo con $\Delta V_2 = 2{,}57V$ e $\Delta V_2 = 2{,}67V$. Trovo T_1 e T_2 e la precisione è $\left|\frac{T_2-T_1}{2}\right|$ (altrimenti derivate....)

La funzione $\Delta V_2(T)$ è una funzione omografica.

Ad alte T c'è un asintoto $\Rightarrow$ valori simili per ampio range di T quindi meno preciso